Integrating synchronization bus timetabling and single-depot single-type vehicle scheduling

Omar J. Ibarra-Rojas, Yasmin A. Rios-Solis
Universidad Autónoma de Nuevo León
Graduate Program in Systems Engineering
Av. Universidad s/n, San Nicolás de los Garza, Nuevo León, México.
Email: [omar,yasmin]@yalma.fime.uanl.mx

Abstract—We address the bus network planning of Monterrey, Mexico, which is similar to those of other cities in Latin America. This city has a large bus network where passenger transfers must be favored, almost evenly spaced departures are sought, and bus bunching must be avoided. We study the timetabling and vehicle scheduling subproblems of bus network planning. By one hand, we use a timetabling formulation with the objective of maximizing the number of synchronizations, i.e., to maximize the arrivals of different bus lines at transfer nodes or to avoid bus bunching along the network. By other hand, we address the single-depot single-type vehicle scheduling with the objective of minimizing the number of vehicles. Since these subproblems are commonly solved in sequential approaches, we propose a bi-objective formulation to consider a more representative problem in bus network planning.

Keywords—Synchronization, timetabling, vehicle scheduling, bi-objective.

I. INTRODUCTION

The entire planning process of a bus network is divided into several subproblems such as line planning, timetable generation, vehicle scheduling, and crew scheduling [2]. Commonly these subproblems are solved in sequential approaches to obtain a solution of the planning problem. However, these methodologies often requires the re-optimization of some subproblems in order to obtain better quality solutions [15]. Particularly, the timetabling and the vehicle scheduling are two subproblems where this problematic is more obvious. The timetabling problem consists in assigning departure times of trips from different lines to achieve a specific goal (commonly based in quality service). The scheduling problem consist in assigning trips of the timetabling to vehicles in order to satisfy a specific goal (commonly based in operational costs). In our case of study, we address the timetabling problem of maximize synchronization, i.e., to favor passenger transfers and avoid bus bunching, and the single-depot single-vehicle type vehicle scheduling problem with the objective of minimize the number of vehicles.

As [18], we also focus on Monterrey, Mexico, where different bus companies share the same demand, and it has a large bus network. Figure 1 shows the entire bus network of Monterrey’s metropolitan area.

The characteristics of the bus network are the presented in [18]:

- The bus network is private and has more than 300 bus lines.
- There is almost full coverage of the territory. The area marked on Figure 1 shows the line concentration downtown since there is a tendency to design lines to cross this area. Therefore, most transfers are located here.
- Some lines have “sub-lines,” which share a common route segment but have variations at the end and beginning of the routes.
- The timetable is only for the company’s administration. Passengers do not know at what time a bus arrives at each stop. They only have an estimate of their waiting-time.

![Fig. 1. Bus network of Monterrey’s metropolitan area. The marked area shows the line concentration downtown [20].](image-url)

In this bus network, the following characteristics must be taken into account.
In several studies (e.g., survey [17]). There are dif-
ferent formulations for every subproblem that corre-
spond to particular characteristics of the bus network. The timetabling problem itself has different formula-
tion corresponding to: frequencies, evenly spaced depar-
ture times, passenger activity-based planning, passenger
transfer at one or multiple stations, and so on.

Passenger transfer is crucial on public transit systems. In several studies ([5], [6], [7], [8], [19], and [24]) the
authors seek to minimize the overall transfer waiting
times and consider evenly spaced departure times. The
minimization of overall waiting time could not be a rep-
resentative objective in our study since we also address
bus bunching. The authors in [21] remark on the neces-
sity of better measurement of the transfer function since
minimizing waiting times may lead to risky passenger
transfers. They make a formulation to minimize the cost
of different kinds of transfers. The authors of [4] address
the problem of synchronizing arrivals of different lines
at some nodes to achieve passenger transfer. They define
synchronization as the simultaneous arrival of two buses
and consider bounds for consecutive trips separation
times (named headway times) and frequency (number
of trips for each line) as given. The objective of the
problem is to maximize the number of simultaneous
arrivals. In [3], the mathematical formulation of the
previous problem is presented. In addition, they show a
constructive heuristic implementation on an Israeli case
study, where they found good solutions within seconds.
The bus bunching problem has been addressed in [10],
where the author develops a dynamic method based
on real-time data to generate holding times for bus
runs to avoid bunching. In [1], the author addresses the
synchronization problem of lines which share common
route segments. He presents a mathematical formulation
to harmonize headway times of different lines to sep-
erate trips in a planning period. He considers possible
departure times as given and solves the problem with an
integrated tabu search and genetic approach using small
computational resources. However, considering different
companies (as it is in our case of study) it might be
difficult to harmonize headway times.

The vehicle scheduling has been addressed in sev-
eral studies and has multiple formulations ([13], [9],
[14], [22]). However, the integration of the VSP with
other planning subproblems is an important research
area. In [23], the authors address the integration of
clock-face timetabling (i.e., services depart at regular
intervals, and thus at the same number of minutes past
each hour) and multiple-depot and vehicle-type vehicle
scheduling problem. They design a tabu search to solve
this integration. However, the idea of the algorithm is
a sequential solution methodology, since timetabling is

- Passenger transfers: Travel from one point to an-
other might imply transfer between lines.
- Bus bunching: There exist sub-lines. Therefore,
different lines commonly converge on a specific
node (called a bus bunching node). Avoiding bus
bunching can be done with trip separation. We
model the separation of trips similar to passenger
transfer in our formulation.
- Almost evenly spaced departures: A large variation
in the time between consecutive trips affects the
behavior of passenger demand, even in small planning
periods.
- Time window synchronization: Two trips are syn-
chronized if the difference between arrival times at
a transfer (or bus bunching) node is within a time
window. These improves interaction between dif-
ferent lines and gives flexibility to the planner. This
flexibility is critical in our case, since companies
do not have bus-control tools, so different travel
times due to drivers’ speeds represent a handicap
for punctual synchronization.

These characteristics of the Monterrey case are also
present in several bus networks in Latin America, in-
creasing the relevance of a flexible model of the synchro-
nization timetabling problem. Therefore, the scheduling
problems has the objective of assigning the trips to the
minimum number of vehicles.

Maximizing line synchronization to reduce bus bunch-
ing and optimize passenger transfers, and minimizing
the number of buses increases the quality of service and
might bring several social benefits such as passengers
switching from cars to public buses, therefore reducing
the particular of vehicles, which represents a needed
energy-sustainable and environmental impact.

On the basis of the above, we propose a bi-objective
formulation of the integrated synchronization timetabling
and single-depot single-type vehicle scheduling problem
to reduce bus bunching, optimize passenger transfer and
minimize number of buses. The remainder of this paper
is organized as follows. First, we present a literature
review in Section II. The formulation of the timetabling
problem and some theoretical results previously made
are presented in Section III. In Section IV, we present
our proposed approach of the integrating formulation of
the timetabling and vehicle scheduling problem. Finally,
we present some relevant aspects and future research in
Section V.

II. LITERATURE REVIEW

The planning of bus networks has been addressed
in several studies (e.g., survey [17]). There are dif-

The planning of bus networks has been addressed
in several studies (e.g., survey [17]). There are dif-
ferent formulations for every subproblem that corre-
spond to particular characteristics of the bus network. The timetabling problem itself has different formul-
tion corresponding to: frequencies, evenly spaced depar-
ture times, passenger activity-based planning, passenger
transfer at one or multiple stations, and so on.

Passenger transfer is crucial on public transit systems. In several studies ([5], [6], [7], [8], [19], and [24]) the
authors seek to minimize the overall transfer waiting
times and consider evenly spaced departure times. The
minimization of overall waiting time could not be a rep-
resentative objective in our study since we also address
bus bunching. The authors in [21] remark on the neces-
sity of better measurement of the transfer function since
minimizing waiting times may lead to risky passenger
transfers. They make a formulation to minimize the cost
of different kinds of transfers. The authors of [4] address
the problem of synchronizing arrivals of different lines
at some nodes to achieve passenger transfer. They define
synchronization as the simultaneous arrival of two buses
and consider bounds for consecutive trips separation
times (named headway times) and frequency (number
of trips for each line) as given. The objective of the
problem is to maximize the number of simultaneous
arrivals. In [3], the mathematical formulation of the
previous problem is presented. In addition, they show a
constructive heuristic implementation on an Israeli case
study, where they found good solutions within seconds.
The bus bunching problem has been addressed in [10],
where the author develops a dynamic method based
on real-time data to generate holding times for bus
runs to avoid bunching. In [1], the author addresses the
synchronization problem of lines which share common
route segments. He presents a mathematical formulation
to harmonize headway times of different lines to sep-
erate trips in a planning period. He considers possible
departure times as given and solves the problem with an
integrated tabu search and genetic approach using small
computational resources. However, considering different
companies (as it is in our case of study) it might be
difficult to harmonize headway times.

The vehicle scheduling has been addressed in sev-
eral studies and has multiple formulations ([13], [9],
[14], [22]). However, the integration of the VSP with
other planning subproblems is an important research
area. In [23], the authors address the integration of
clock-face timetabling (i.e., services depart at regular
intervals, and thus at the same number of minutes past
each hour) and multiple-depot and vehicle-type vehicle
scheduling problem. They design a tabu search to solve
this integration. However, the idea of the algorithm is
a sequential solution methodology, since timetabling is
modified and then, optimize the vehicle scheduling problem. In [15], a similar sequential idea was presented. The authors address the timetabling problem with objective of minimizing the overall waiting time considering a given frequency and fixed headway times. They design an iterated local search for solving their formulation. In [12], the authors propose a measure function for transfers based on ideal waiting times. They design an optimization approach to minimize other vehicle costs such as number of vehicles and unproductive time. In [11] and [16], an integral formulation for timetabling and vehicle scheduling that considers weights on the objective function is presented. However, these weights reflect the planner’s necessity, which is a complex characteristic if two or more objectives are in conflict.

The literature review presents different characteristics of the bus network planning subproblem. However, to the best of our knowledge, there is not an integrated formulation of STP and VSP that represents all the characteristic of our case study, which is similar to some cities in Latin America.

III. TIMETABLING INTEGER PROGRAMMING FORMULATION

In this section, we present the integer programming formulation of the Bus Network Synchronization Timetabling Problem (BTP) according to [18].

There are two types of key nodes: transfer and bus bunching nodes. Figure 2 shows these two cases. Case (a) represents two lines, i and j, which converge at node b and have a common line segment. Avoiding bus bunching of these lines implies the separation of their trips on node b. This represents the case of sub-lines, where for example, the time between two buses is of 5 minutes to avoid bunching. Case (b) represents a transfer node where many passengers would like to go from trips on line i to trips on line j. A reasonable waiting-time for the passenger transfers on this type of node is needed, for example, 3 minutes.

![Diagram](image)

Fig. 2. Two types of synchronization nodes on the bus network. Case (a) represents a bunching node and Case (b) represents a transfer node [18].

A. Sets

We use the following sets to represent the elements of the formulation.

- \( I \): Set of lines of the bus network.
- \( J(i) \): Set of lines that may have synchronization nodes with line \( i \in I \).
- \( B^{ij} \): Synchronization nodes for pair of lines \((i,j)\).

B. Parameters

Our assumption is that the following data is available.

- \( T \): Planning period in minutes. These can be divided in type of day (holiday, weekend, etc) and demand behavior in order to obtain more quality parameters.
- \( f_i \): Frequency (number of trips) of each line \( i \in I \).
- \( t_{ij} \): Travel times of each line \( i \) to each synchronization node \( b \).
- \([w_{b}, W_{b}]\): Waiting-time window for each synchronization node \( b \).
- \( H_{\text{ideal}} \): Separation time between consecutive trips of line \( i \) called headway time.
- \( \delta \): Relative tolerance of headway times of line \( i \).

On the basis of the ideal headway time, we define the minimum and maximum headways times of line \( i \) as \( [h^i, H^i] = [H_{\text{ideal}} - \delta, H_{\text{ideal}} + \delta] \).

C. Decision variables

Since we have to determine the departure time and identify the synchronization of different pairs of trips, we propose the following decision variables.

- \( X_p^i \): (integer) departure time of the \( p \)-th trip of line \( i \).
- \( Y_{pqb}^{ij} \): 1, if the \( p \)-th trip of line \( i \) arrives first at node \( b \) and synchronizes with the \( q \)-th trip of line \( j \), and 0, otherwise.

D. Formulation of STP

The bus synchronization timetabling problem (STP) is given by

\[
\max F_{\text{STP}}(Y) = \sum_{i \in I} \sum_{p=1}^{f_i} \sum_{j \in J(i)} \sum_{q=1}^{f_j} \sum_{b \in B^{ij}} Y_{pqb}^{ij}
\]
subject to

\begin{align}
X^i_1 & \leq H^i \\
T - H^i & \leq X^i_f, \leq T \\
h^i & \leq X^i_{p+1} - X^i_b \leq H^i \\
(X^i_q + t^i_b) - (X^i_p + t^i_b) & \geq w_b - M(1 - Y^i_{pqb}) \\
(X^i_q + t^i_b) & \leq W_b + M(1 - Y^i_{pqb}) \\
Y^i_{pqb} & \in \{0, 1\} \\
X^i_b & \in \{0, 1, \ldots, T\}
\end{align}

The objective function maximizes the number of synchronizations. Constraints (1) force the first trip of each line \(i\) to depart at the beginning of the planning period \(T\). Similarly, constraints (2) force the last trip of each line \(i\) to depart at the end of the planning period \(T\). Constraints (3) separate consecutive trips of each line \(i\) by the number of minutes between the minimum \(h^i\) and maximum \(H^i\) headway time. Constraints (4) and (5) are used by the objective function to activate the synchronization variables \(Y^i_{pqb}\) if the difference between arrivals of the \(q\)-th trip of line \(j\) and the \(p\)-th trip of line \(i\) at node \(b\) is between \(w_b\) and \(W_b\). Although \(M\) is a big number, it is bounded by the maximum difference of arrival times of every pair of lines \((i, j)\) that synchronize at every node \(b\), that is, \(M = \max_{i,j \in A(p), b \in B} \{(T + t^i_b) - t^j_b\}\).

Finally, constraints (6) and (7) represent the domain of the decision variables.

In [18], some theoretical results were presented about this problem. First the NP-hardness of the problem was proved, and also a preprocessing stage to eliminate decision variables was developed. In this preprocessing stage a feasible departure window \(D^i_p\) was calculated for all trip \(i \in I, p = 1, \ldots, f^i\) as follow. Constraints (1) and (2) restrict the first and last trip of each line \(i\) to depart at the beginning and at the end of the planning period \(T\), respectively. Constraints (3) separate consecutive trips of each line \(i\) by a valid headway time; using this headway time window, it can be determine a minimum and maximum departure time of every trip \(p\) of line \(i\) that may satisfy constraints (1) and (2), i.e., a departure time window \(D^i_p\) for every trip \(p\) of line \(i\), where \(X^i_p\) might satisfy constraints (1), (2), and (3) if \(X^i_p \in D^i_p\).

Figure 3 shows the construction of the departure time window of the eighth trip \((p = 8)\) of line \(i\). The example instance has a planning period \(T = 30\) minutes, a frequency \(f^i = 10\), and relative tolerance \(\delta^i = 0.33\%\), i.e., \(h^i = 2\) and \(H^i = 4\). Figure 3 has four time lines. The first line represents the earliest departure time of the eighth trip, assuming that the first trip departs at \(X^i_1 = 0\). The second line represents the latest departure time of the eighth trip, assuming that the first trip departs at \(X^i_1 = H^i\). The third line represents the earliest departure time of the eighth trip, assuming that the last trip departs at \(X^i_f = T - H^i\). The fourth line represents the latest departure time of the eighth trip, assuming that the last trip departs at \(X^i_f = T\). Therefore, the intersection of the earliest and latest departure times \([\max\{14, 18\}, \min\{30, 26\}]\) results in the feasible departure time window \(D^i_8\).

![Figure 3. Feasible departure time window \(D^i_8\) for the eighth trip of line \(i\) corresponding to an instance with \(T = 30\), \(f^i = 10\), and \(\delta^i = 0.33\%\) [18].](image)

Generally, the departure time window \(D^i_p\) for every \(p - th\) trip of line \(i\) is between \(\max\{h^i(p-1) + T - (f^i - (p-1)H^i)\}\) and \(\min\{H^i + T - (f^i - p)H^i\}\).

If we consider the departure time window \(D^i_p\) of each trip, an arrival time window \(A^i_{pb}\) of each trip \(p\) of line \(i\) at node \(b\) can be defined by shifting \(D^i_p\) by \(t^i_b\) time units. Therefore, the arrival window \(A^i_{pb}\) is between \(\max\{h^i(p-1) + T - (f^i - (p-1)H^i)\} + t^i_b\) and \(\min\{H^i + T - (f^i - p)H^i\} + t^i_b + W_b\).

Similarly, there exist a synchronization window \(S^i_{pb}\) for each trip \(p\) of line \(i\) and node \(b\) between \(\max\{h^i(p-1) + T - (f^i - (p-1)H^i)\} + t^i_b + W_b\) and \(\min\{H^i + T - (f^i - p)H^i\} + t^i_b + W_b\).

In [18], it was proved that for any trips \(p\) and \(q\) of lines \(i\) and \(j\), respectively, and any synchronization node \(b\) of BTP, \(S^i_{p} \cap A^j_{q} = \emptyset\) if and only if, \(Y^i_{pqb} = 0\) due to feasibility, and constraints (4) and (5) related to this indices are redundant. Using this result, it can be eliminated up to 70% of synchronization variables and constraints of the formulation. And constraints (1),(2), and (7) can be replaced by \(X^i_p \in D^i_p\).

IV. INTEGRATING VEHICLE SCHEDULING

In our case, each line has one type of bus and one depot to start the route. Then, we integrate the
timetabling problem with the single-depot single-type vehicle scheduling problem. This problem can be formulated as a network flow problem in a bipartite graph. Figure IV shows a graph with \( n+2 \) nodes. Node \( s \) represents the depot when a bus depart to its first trip, node \( t \) represents the depot when a bus finishes its schedule, and the rest of nodes are the trips of the timetabling. There exists an arc \((i, j)\) if it is possible to make trip \( j \) after trip \( i \) using the same vehicle. Then, the objective is minimizing the number of vehicles, which are represented by the minimum number of paths from \( s \) to \( t \) restricting all nodes to be assigned to only one path (vehicle).

Fig. 4. Bipartite graph representing the single-type single-depot vehicle scheduling problem [13].

A. Parameters

We need specific parameters to integrate the VSP. This parameters are represented as follows.

- \( r^i \): Turn around time for line \( i \).
- \( l^i \): Number of vehicles for line \( i \).

B. Decision variables

We could use the departure times to define the VSP network in sequential approach. However, we can do the same adding a variable to define the existence of an arc between two trips. To achieve this, we use the following variables.

- \( Z^i_{pq} \): 1, if it is possible to assign \( q \)-th trip after \( p \)-th trip of line \( i \) to the same vehicle, and 0, otherwise.
- \( V^i_{pq} \): 1, if \( q \)-th trip is assigned after \( p \)-th trip of line \( i \) to the same vehicle, and 0, otherwise.

C. Integrated STP and VSP formulation (STVSP)

For the formulation, we add two artificial trips 0 and \( f^i + 1 \) for all line \( i \), to represent the starting and ending node of a vehicle schedule. Then, the objective, i.e., the number of vehicles in the VSP can be represented as follows.

\[
F_{VSP}(V) = \sum_{i \in I} \sum_{p=1}^{f^i} V^i_{0p}
\]

Therefore, the bi-objective formulation of STVSP is given by:

\[
F(X, V) = [\max F_{STP}(X), \min F_{VSP}(V)]
\]

subject to (3)-(6)

\[
X^i_p - X^i_p \geq r^i - r^i(1 - Z^i_{pq}) \quad (8)
\]

\[
V^i_{pq} \leq Z^i_{pq} \quad (9)
\]

\[
\sum_{q>p} V^i_{pq} = \sum_{q<p} V^i_{qp} = 1 \quad (10)
\]

\[
\sum_{p} V^i_{0p} = \sum_{p} V^i_{p(f^i+1)} \leq l^i \quad (11)
\]

\[
Z^i_{pq} \in \{0, 1\}, V^i_{pq} \in \{0, 1\}, X^i_p \in D^i_p \quad (12)
\]

Where constrain (8) specify the construction of the bipartite graph, since the arc \((p, q)\) exist for line \( i \) only if \( Z^i_{pq} = 1 \). Then, constraints (9) indeed indicate if \( q \)-th trip is assigned after \( p \)-th trip of line \( i \) for some vehicle. The expressions (10) are classic conservation flow constraints. The set of constraints (11), force that the number of vehicles used is less or equal than the number of vehicles available.

We propose this bi-objective formulation in order to attack the integration problem more accurately. For example, in [16] an integration of timetabling and vehicle scheduling problem is presented, but the author uses a weighted sum of problem objectives, therefore, the weight parameters representativeness is needed. This could be difficult to obtain in a bus network as the presented in this study. So using the bi-objective approach we will be able to propose a set of solution to the planner.

V. DISCUSSION AND FUTURE RESEARCH

The formulation of the timetabling problem proposed in this study, is more representative than other formulations existent in literature for the Monterrey’s bus network. Some important characteristics are the synchronization within a time window (rather than a single moment) and the relative tolerance in the headway times, in order to make a flexible model. The advances in forecasting travel times on the transit network based on historical data and real-time bus-control tools can greatly improve on the precision of the timetabling model and the quality of the solutions, i.e., the quality of the bus
network’s service. On the other hand, the bi-objective integration with the VSP, represent a new approach to subproblems integration, and considering that optimizing one subproblem could lead to bad quality solutions in subsequent subproblems, it seem accurate. The idea of solution algorithms for the VSP could be explode in the STVSP to design an efficient solution algorithm. However, it is necessary to design a methodology to attack the timetabling problem since its intractability remains, even with the preprocessing stage.

The aspects that should be considered for further studies are the following:

- In real cases, it is necessary to integrate passenger demand behavior to determine more accurate planning periods. Also, forecasting travel times might bring important improvements to the formulations representativeness.
- Of course, an efficient solution algorithm is needed. We will attack the timetabling problem first since the intractability of this problem remains. Then, we can design efficient solution algorithms for the bi-objective formulation.
- To analyze other VSP formulations which may bring benefits to the integrated approach. For example, in [22] a formulation that considerably reduces the size instances was developed.

ACKNOWLEDGMENTS

This study was partially supported by Grants C003-2009-01/113032 and 101857 awarded by CONACYT. We thank DAS-SISTEMAS, Fernando López, Ada Álvarez, and Paulina Ávila for the fruitful discussions. Finally, thanks to María Angélica Salazar for her valuable comments.

REFERENCES