Heuristics for a Stochastic and Dynamic Routing Problem in Industrial Shipping

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Abstract—Vehicle routing problems are an essential element in transportation planning and logistics. In particular, maritime transportation plays a crucial role in international trade, managing most of long-distance shipments. Static and deterministic vehicle routing problems have been extensively studied in the literature and meta-heuristics have shown to be capable of providing near optimal solutions for most of them. However, stochasticity is inherent in numerous important real applications, many of which are also dynamic, and this area is significantly more unexplored yet. In this direction, this paper approaches a dynamic and stochastic maritime transportation problem motivated by a real world application in industrial shipping. Three heuristics adapted to this problem are considered and their performance minimizing transportation costs is evaluated. Extensive computational testing to further highlight properties of the considered solution methods is provided and it shows that the use of stochastic information in the solution methods yields to an average cost saving of 2.5 % on a set of realistic test instances.

Keywords—Maritime transportation, Scenario, Simulation, Tabu Search.

I. INTRODUCTION

Maritime transportation is a significant component in international trade, and more than 7 billion tons of goods are carried by ship annually [18]. The costs related to ship can be very high, with daily time-charter rates and fuel costs that can amount to tens of thousands of USDs. Proper planning of routes and schedules is crucial for shipping companies in order to achieve a good fleet utilization and reduce costs. Research on ship routing and scheduling has therefore experienced increased interest during the last decades, see for example the reviews by [16], [17] and [5], [4].

One usually distinguishes between three different modes of maritime transportation: liner, tramp, and industrial [13]. In the latter the ship operator owns or controls both the cargo to be transported and the ships performing the transportation. The focus of the operator is therefore to minimize the total transportation costs while ensuring that all cargoes are transported. A cargo consists of a specified quantity of a given product that must be picked up at its port of loading, transported and then delivered to its port of discharge. There are specified time windows during which the loading of the cargoes must start, and there may also be time windows for discharging. The operator controls a heterogeneous fleet of vessels, with different sailing speeds, cost structures, and loading capacities. There are two ways of transporting cargoes; the operator may either transport a cargo using a ship from its own controlled fleet, or may hire the services of other shipping companies. For the latter we assume that the operator will use spot carriers, which implies that a lump sum is paid for the transportation of a given cargo.

Planning of ship routes and schedules in industrial shipping, as well as in the other transportation modes, is to an increasing degree performed with assistance from optimization based decision support systems, as illustrated by [7] and [8]. In practice, this involves using heuristic solution methods to solve deterministic optimization problems based on known information. This is done on a continuous basis as new relevant information, such as the occurrence of a new cargo, arrives, or at given time intervals. Most literature on ship routing and scheduling also solve static and de-
terministic versions of the problem. However, previous work on dynamic and stochastic versions of land-based transportation planning problems has indicated that the inclusion of stochastic information in such a dynamic planning process is valuable, see for instance the work by [2] and [9], [10]. The purpose of this paper is to develop and test alternative solution methods for a stochastic and dynamic routing problem in industrial shipping. Considering that maritime transportation represents a major factor in overall freight transportation and the high costs related to it, there is an economic incentive to test if similar methodologies can be adapted to the maritime case. We also aim at testing which types of industrial ship routing and scheduling problems that can gain the most from solving them using stochastic information.

In this paper, discrete event simulation is used to reproduce the planning environment and to handle generation of new cargo requests over time. Whenever a replanning action is scheduled, heuristics are run to produce solutions consistent with currently available information. Three different heuristics are considered. In the first heuristic, called the myopic dynamic heuristic (MDH), a deterministic subproblem is solved using an adaptation of the tabu search presented by [12]. This method does not utilize any stochastic information. The second heuristic is based on the multiple scenario approach (MSA) of [2] and relies on generating a set of scenarios that consists of the currently known cargoes plus a sampled realization of the future cargoes. Each of these scenarios are solved using the tabu search, and one of the solutions produced are selected based on a consensus function; the chosen solution is that which is the most similar to other solutions. The third heuristic that we suggest is an adaption of the branch-and-regret heuristic (BRH) of [10]. Again scenarios are generated and solved using tabu search. Rather than selecting one of the resulting solutions, an iterative procedure is followed where parts of the solution are gradually fixed in each of the scenarios until all scenarios have converged to the same solution. We test the three heuristics on a number of realistic, but randomly generated test instances.

The remaining parts of this paper are organized as follows: A literature review is presented in Section II. Section III gives a description of the industrial ship routing and scheduling problem, including a mathematical formulation of the static and deterministic version of the problem. In Section IV we present the three heuristics, as well as the simulation procedure and the tabu search that is used in all three heuristics. A computational study is presented in Section V, while concluding remarks are given in Section VI.

II. RELEVANT LITERATURE

Most literature on ship routing and scheduling involves solving static and deterministic problem variants. Two counterexamples that consider operational routing between two points can be mentioned: [14] attempt to minimize the expected fuel consumption by exploiting uncertain ocean currents and [1] represent weather conditions in a stochastic dynamic network.

Considering the tactical level, [3] develop a decision support system for a crude oil transportation and inventory problem that takes into account demand uncertainty and travel time uncertainty. As opposed to the work presented here, the demand uncertainty is with respect to the rate of consumption rather than the appearance of new cargo requests. As [3] consider only one production port, the problem does not have the same pick-up and delivery structure as in our problem, and only a small set of possible routes are considered from the production port to consumption ports. The crude oil transportation and inventory problem is formulated as a discrete time Markov decision process and solved heuristically. [11] consider the risk associated with fluctuating spot rates and seek to maximize the profit while constraining the variance. The problem is relevant in both tramp and industrial shipping, as optional cargoes are included. Two different formulations are presented, and one of them is solved using a branch-and-price-and-cut method. Only full-load instances are considered, meaning that only one cargo can be onboard any ship at any time, and the problem is considered as a static problem. Uncertainty is associated to future prices and revenues for voyage charter opportunities.

III. PROBLEM DESCRIPTION

A static and deterministic industrial ship routing and scheduling problem can be described as follows, building on a formulation provided by [4]. The objective is to minimize total variable costs, while transporting all cargoes using either the fleet of the company or by using spot voyages. Each cargo has a given loading port and unloading port, and the loading and unloading both have an associated time window within which service can take place. A ship may be able to carry several cargoes simultaneously, but the total load cannot violate the ship capacity.

In a mathematical formulation, let $i$ denote a cargo. Cargo $i$ is associated to a loading node $i$ and an unloading node $N + i$ where $N$ is the total number of cargoes. The set of loading nodes is denoted by $N^P = \{1, \ldots, N\}$, and the set of unloading nodes is given as $N^D = \{N+1, \ldots, 2N\}$. Let $v$ denote a ship, and take $V$
as the set of all ships in the fleet of the shipping company. Each ship has a node $o(v)$ representing its origin, and a node $d(v)$ representing its final destination. Let $N_v$ be the subset of all nodes which are compatible with ship $v$, meaning that ship $v$ is allowed to visit the node and to carry any cargo associated with the node. Furthermore, define $N^P_v = N^P \cap N_v$ and $N^D_v = N^D \cap N_v$. Let $A_v \subseteq N_v \times N_v$ be the set of arcs $(i,j)$ such that the ship can sail directly from the port associated with node $i$ to the port associated with node $j$. Sailing from node $i$ to node $j$ using ship $v$ has a variable cost $C_{ijv}$ and a total sailing time $T_{ijv}$. If the cargo is serviced using a spot voyage, the associated cost is $C^S_{iv}$. The ship capacity is given as $Q^V_i$, and the size of each cargo is given as $Q^N_i$ for $i \in N_P$. The time window of a cargo $i$, if serviced by ship $v$, is given as $[E_{iv}, L_{iv}]$.

To formulate a mathematical program, decision variables are introduced with binary flow variables $x_{ijv}$ being 1 if ship $v$ sails from node $i$ to node $j$ and being 0 otherwise. Continuous time variables $t_{iv}$ represent the time at which service begins at node $i$ if serviced by ship $v$, and continuous variables $l_{iv}$ gives the total load onboard ship $v$ after completing service at node $i$. Binary variables $y_i$ are 1 if cargo $i$ is taken by a spot voyage and 0 otherwise. The objective function and constraints can now be stated.

The objective function (2) sums all variable costs associated to sailing the company’s own fleet plus the cost of using spot voyages. Constraints (3) ensure that each cargo is transported, either by a company controlled ship or by using a spot voyage. Flow conservation constraints are expressed through (3)–(5). Time variables are controlled through constraints (6) and (7); the former makes sure that travel times are added between consecutive visits on a ship route, whereas the latter ensure that all visits are made within the time windows for service. The big constant in (6) $M_{ijv}^T = L_{iv} + T_{ijv} - E_{ijv}$. Constraints (8) state that the ship is empty when it leaves the origin node. When visiting a loading node, constraints (9) make sure that the load is increased at least by the size of the cargo transported. Correspondingly, constraints (10) force the load to be reduced by not more than the size of the cargo delivered. Together, constraints (9) and (10) ensure that $l_{iv}$ becomes a lower bound on the amount carried by the ship at any time, thus enforcing capacity restrictions to hold when coupled with constraints (11) and (12). For each cargo, loading and unloading nodes must either be serviced by a spot voyage or by the same ship, which is enforced through constraints (13). If serviced by the same ship, the loading node must be serviced before the unloading node, in accordance with constraints (14). Finally, the domain of the variables are given by (15)–(18).

The above formulation describes a static and deterministic problem, whereas the problem studied here is dynamic and stochastic. Using the definition by [15], a problem is dynamic if inputs to the problem become known to the decision maker at the same time as the solution is determined. For the industrial ship routing and scheduling problem above, this means that information about new cargoes (including information about loading and unloading ports, time windows, and sizes) will arrive while determining the routes for the ships in the fleet. For an industrial shipping company, the problem is open-ended, that is, new information about cargoes will continue to arrive as long as the company is operating. When describing the problem as stochastic, this refers to the fact that new cargoes will arrive based on stochastic processes which can be exploited in the decision making.

It is reasonable to believe, in the context of industrial shipping, that historical data and production forecasts can be used to derive probability distributions on arrivals of cargoes and their properties.

## IV. Solution methods

This section describes solution methods for the dynamic and stochastic version of the industrial ship routing and scheduling problem of Section III. To evaluate solution methods, the operations of the shipping company are simulated over a long time horizon. Details of this simulation can be found in Section IV-A. During the simulation decision points arise, where the company is allowed to replan the ship routes and schedules. Three different methods to perform this replanning is considered; the myopic dynamic heuristic is described in Section IV-B, the multiple scenario approach with consensus is described in Section IV-C, and the branch-and-regret heuristic is described in Section IV-D. When performing replanning, each of the three methods mentioned above relies on solving instances of the static and deterministic version of the problem described in Section III. To do this, they make use of a tabu search which is described in Section IV-E.

### A. Simulation

In a dynamic problem information is revealed over time. The process of making decisions and receiving new information is represented in a discrete event simulation. The state of the simulation system consists of all available information about cargo requests and vessels, as well as the current plan containing routing and scheduling decisions. Events that are treated in the simulation include arrival of new cargo requests and vessels arriving...
a port. The simulation clock can be updated using either fixed time increments or time increments based on the next event. Whenever the simulation clock is updated, whether with a fixed time increment or an increment such that the next event is reached, the information about vessels and cargoes is updated and one of the three methods described in Sections IV-B–IV-D is used to update the current routes.

When the simulation starts, we assume that all vessels are empty and waiting at a port. Some cargoes are known, and a first plan is made. The simulation then continues for 360 days, after which key data, such as total cost incurred, are calculated. Based on the fact that shipping companies are very interested in maximizing the utilization of their own fleet, vessels depart to their next destination immediately after finishing a loading or unloading operation at a port. To make a fair comparison of the three solution methods used to make decisions, the end of the time horizon must be handled with care. The solution methods are provided with a set of sample scenarios that include stochastic information, but no stochastic information is included beyond the end of the simulation. Furthermore, when no new events will take place within the simulation, the plan with the remaining cargoes is made by solving a static and deterministic version of the industrial ship routing and scheduling problem. As long as the event queue of the simulation is not empty, one of the solution methods in Sections IV-B–IV-D is used to create the current set of routes.
The myopic dynamic heuristic (MDH) is a simple heuristic that can be used on dynamic and stochastic transportation problems. It considers the problem as a purely dynamic problem, using only information that is known with certainty and completely ignoring stochastic information concerning future events. Whenever replanning is allowed, a subproblem is considered that consists of all known cargoes. This subproblem is a static and deterministic version of the problem described in Section III, and is solved directly using the tabu search procedure described in Section IV-E. The solution of each stage is used as initial solution for the following stage, after removing all cargoes already served and adding the cargoes that became known during the last stage. The cargoes that were loaded but not unloaded are locked to their corresponding vessel, so that they cannot be reassigned to a different vessel during the search process.

The MDH may be considered as a simple benchmark, which is very similar to current practice among shipping companies that use decision support systems, in that a static and deterministic problem is considered every time a new plan is made. The MDH has only one parameter, given that the tabu search used to solve the subproblem has already been tuned: one simply has to determine how many iterations the tabu search should run for each subproblem encountered. An overview of the MDH is shown as Algorithm 1.

Algorithm 1 Myopic dynamic heuristic
1: for each point in time where replanning is allowed do
2: Formulate an instance of the static and deterministic problem (1)-(18), where variables corresponding to decisions that have already taken place are fixed.
3: Solve the instance using the tabu search of Section IV-E.
4: Use the solution as the new plan for the overall problem.
5: end for

The consensus function used is defined as follows. Having generated a solution for each sample scenario, define a two-dimensional matrix $M \equiv (M[v,c])$ where $M[v,c]$ represents the number of scenario solutions for which cargo $c$ is served first by vessel $v$. Then, if the scenario solutions are denoted by $s_1, \ldots, s_k$ and the cargo served first by vessel $v$ in solution $s_i$ is denoted by $c_i^v$, the considered consensus function $f$ is defined as $f(s_i) = \sum_{v \in V} M[v,c_i^v]$, which represents the score of scenario solution $s_i$ according to the consensus evaluation. Then, the scenario solution with the highest score is used to build the final plan for the current stage. To complete the final plan, sampled cargoes are first removed from the chosen sample scenario solution.

The MSAC has three important parameters: the number of iterations used in the tabu search, the number of sample scenarios used, and finally one parameter controlling the number of sampled cargoes to be included in each scenario. Since the problem is open-ended there is no natural point in time that can be used to stop generating sampled cargoes. It is necessary to include a large enough number of sampled cargoes to properly represent the potential effects of future cargo requests on the routing and scheduling decisions. At the same time, avoiding too many sampled cargoes is necessary so that
the subproblems do not become too difficult and time consuming to handle for the tabu search. A parameter is therefore used to specify the number of sampled cargoes to generate as a percentage of the current number of cargo requests known with certainty. The MSAC is summarized as Algorithm 2.

Algorithm 2 Multiple scenario approach with consensus function

1: for each point in time where replanning is allowed do
2: Generate $k$ sample scenarios.
3: Formulate $k$ instances of the static and deterministic problem (1)–(18), where variables corresponding to decisions that have already taken place are fixed and where the cargoes included correspond to those present in the sample scenarios.
4: Solve the instances using the tabu search of Section IV-E.
5: Calculate $M[v, c]$, for each combination of vessel $v$ and known cargo $c$.
6: Find the scenario $i$ that maximizes the consensus function $f(s_i)$.
7: Remove all sampled cargoes from the solution, and use as the new plan for the overall problem.
8: end for

D. Branch-and-Regret Heuristic (BRH)

The branch-and-regret heuristic (BRH), introduced in [10] as an improvement of the dynamic stochastic hedging heuristic of [9], is another heuristic designed to take advantage of stochastic information that is available when solving dynamic and stochastic transportation problems. As in the case of MSAC, this is done by using sample scenarios generated from the stochastic information of the problem. Rather than solving a problem for each scenario once and then choosing one of the solutions, the BRH works by iteratively making decisions about the new plan and solving for each scenario to evaluate the decisions. That is, the BRH starts by solving all scenario problems separately, but in these solutions decisions regarding known customers will most likely end up being different due to the presence of sampled customers. Conceptually, the BRH identifies structural decisions and branches on them until all scenario problems have solutions where all known customers are serviced according to the same plan.

In detail, at each decision point reached during the simulation, the BRH is divided into two phases as follows. First, all scenario subproblems are solved individually using tabu search. Then, the frequency of assigning cargo $c$ to vessel $v$ in the solutions is recorded for every pair of known customer $c$ and vessel $v$. The pair $(c, v)$ with the highest frequency is chosen and then two branches are created, one forcing cargo $c$ to be served by vessel $v$ and another one forbidding that to happen. Solutions for all scenarios are found for both branches and the one with the smallest overall cost (averaged over all scenarios) is chosen. If the first branch is selected, cargo $c$ is locked to vessel $v$ for the remaining iterations. On the contrary, if the second branch is chosen, cargo $c$ is declared incompatible with vessel $v$, and this is indicated in the compatibility matrix for the remaining iterations. This phase stops when all cargoes are locked to some vessel or there are no unlocked cargoes compatible with more than one vessel.

After finishing the first phase, each cargo is assigned to a specific vessel, but each scenario solution may provide a different sequence for each vessel (containing the same cargoes). That is, for each vessel $v$ several sequences of known customers may be obtained by ignoring sampled cargoes from the solutions of the scenarios. In practice, the number of different sequences will be small due to the presence of time windows. The second phase focuses on determining which sequence (or route) to use for each vessel. This selection is performed as follows. First, the list of vessels is sorted according to the frequency of the most frequent sequence over the set of scenarios, and the vessels are considered following that order. For a given vessel, each available sequence is considered and solutions to all scenarios are found while using that particular sequence, obtaining an evaluation of the sequence. Then the sequence with the best overall evaluation is selected and used for its corresponding vessel. This decision process is performed for each vessel until there are no more vessels remaining. After finishing the second phase a plan is obtained where each cargo is assigned to a vessel and each vessel follows a particular route. At this point, all sample scenarios have solutions wherein the same plan is followed, and the average cost of the sample scenarios gives some indication of what will be the future total cost of performing the routes. An overview of the BRH is shown as Algorithm 3.

E. Tabu Search

To solve the deterministic subproblems arising every time replanning is to be performed we adapted the fast tabu search heuristic for ship routing and scheduling of [12], which in turn is based on the unified tabu search algorithm of [6]. Since the subproblems we consider are solved repeatedly, some simplifications are made to speed up the execution of the tabu search. Additionally,
Algorithm 3 Branch-and-regret heuristic

1: for each point in time where replanning is allowed do
2:  Generate $k$ sample scenarios.
3:  Formulate $k$ instances of the static and deterministic problem (1)–(18), where variables corresponding to decisions that have already taken place in reality are fixed and where the cargoes included correspond to those present in the sample scenarios. Also fixed are variables corresponding to decisions selected during the BRH, including that some cargoes are forced to be visited by some vessels, and some cargoes are prohibited from being visited by some vessels.
4:  Solve the instances using the tabu search of Section IV-E.
5:  Count the frequency of cargo $c$ being serviced by vessel $v$.
6:  Consider all pairs $(c, v)$ on which the heuristic has not previously branched, and select the one with highest frequency. If the frequency is 0 go to Step 8.
7:  Solve all scenario problems again, so that solutions are obtained both for the case where cargo $c$ is serviced by vessel $v$ and where cargo $c$ is serviced by a different vessel. Calculate the average cost of solutions representing both those decisions and enforce from here on the decision associated to the lowest cost. Go back to Step 3
8:  For each vessel, make a list of all sequences in which known customers are being serviced. Order the vessels according to the frequency of their most used sequence.
9:  Consider the vessel for which a sequence has not yet been determined that has the most frequently used sequence. Solve all scenario problems again, to obtain a solution for each choice of sequence. Calculate the average cost of scenario solutions for each sequence and enforce from here on the sequence associated to the lowest cost.
10: If some vessels remain where different sequences are used in the scenario solutions, go to Step 9
11: Consider the final solution where all sampled cargoes are disregarded, and use this as the new plan for the overall problem.
12: end for

Our tabu search incorporates some additional features that are necessary to handle the specific ship routing problems arising as subproblems in the BRH.

New instances of the static and deterministic problem arise as the simulation proceeds. In the new instances, some new cargoes may have been added and some previously known cargoes may already be picked up or delivered. The initial solution for the new instance is created by modifying the best solution from the previous planning period, disregarding cargoes that have already been delivered and otherwise fixing parts of the routes corresponding to actions already performed. Thus, deliveries corresponding to cargoes already picked up are not allowed to be re-scheduled. The option of using spot voyages to transport cargoes is handled by a dedicated dummy ship. All cargoes in the new instance that were not part of the solution for the previous planning period is initially allocated to the dummy ship, and in particular, for the very first instance in the simulation all cargoes are initially placed on the spot voyage dummy ship.

At each iteration of the tabu search, the best neighbor of the current solution is selected as the current solution for the next iteration. The neighborhood of a solution $s$ is formed by all solutions that can be obtained from $s$ by moving one cargo from one vessel to another. Once the cargo to be moved and the new vessel are fixed, the loading and unloading positions for that cargo must be determined. To reduce the size of the neighborhoods considered, first the best loading position is found separately, and then the best unloading position is determined subject to the fixed loading position. The data concerning these moves is stored to be used in subsequent iterations and avoid unnecessary recalculations. Once a move is made, moving cargo $j$ from vessel $v$, it will be considered tabu to move the cargo back to the same vessel. The tabu status is upheld for a fixed number of iterations during which moves involving cargo $j$ on board vessel $v$ are prohibited. An aspiration criterion applies, so that the tabu status is relaxed for moves leading to a solution that is better than the best solution found so far.

The tabu search includes a relaxation mechanism to allow the consideration of intermediate infeasible solutions violating capacity or time window constraints. These violations are penalized in the move evaluation function, and the weights associated to these penalties are updated dynamically. If we denote the capacity and time window violations of a solution $s$ as $q(s)$ and $w(s)$, respectively, a solution $s$ is evaluated according to the function:

$$f(s) = c(s) + \alpha q(s) + \beta w(s) + p(s)$$
where \( c(s) \) is the cost of the solution (transportation and spot voyages costs plus port fees) and \( \alpha \) and \( \beta \) are the weights (real positive values) of the violations. The term \( p(s) \) refers to a dynamic, solution dependent penalty that is added to diversify the search. The details of this penalty can be found in [12].

Since the static and deterministic version of the ship routing and scheduling problem must be solved frequently, it is important that the execution of the tabu search is as fast as possible. Thus, we do not consider the periodic route reoptimization process and the final intensification phase used in [12]. We employ the same parameter settings as used in [12], except that the update factor of \( \alpha \) and \( \beta \) and the diversification intensity are kept fixed during the search. A problem feature that is distinct for problem instances encountered in the BRH is that certain cargoes may be either fixed to a specific vessel or prohibited from being served by a specific vessel. To allow these conditions to be imposed a compatibility matrix is introduced, stating which vessels can be used for each cargo, and only moves verifying the conditions of the compatibility matrix are considered in the search.

V. Computational study

The computational experiments have two parts. First we calibrate the solution methods on a small set of instances (Section V-A). Second we run the methods on a set of 96 realistic instances with varying characteristics (Section V-B). The instances that have been generated vary based on the fleet size (small sized fleet with 6 vessels, medium size fleet with 12 vessels or big sized fleet with 24 vessels), the fleet composition (heterogeneous, with vessels with different cost and capacity, or homogeneous), the size of cargoes (full load or parcel load), the number of cargoes (high or low demand), and the number of ports (16 or 32). Each instance is simulated over a one year horizon.

A. Calibration

In Figure 2 we calibrate the number of iterations to use per subproblem for the MDH. At the same time we test different strategies for updating the time during the simulation: either based on events such as arrival of new cargoes and arrivals at ports, or using a fixed time increment of two, four, eight, or sixteen days. The numbers on the secondary axis have the following meaning: each data point consists of the average quality over the five instances used in the calibration, where the quality is normalized to give a number in \([0, 1]\) for each instance. Within a single set of tests, the best solution has a quality of 1 and the worst solution a quality of 0. This is repeated for each of Figures 2–6. For the MDH, using a fixed interval time update is not worse than using event based updates, although large steps seem worse than small steps. To facilitate comparisons with other solution methods, where MSAC is built around using event based updates, all three methods will use the event based updates in the following. It is also clear that increasing the number of tabu search iterations per subproblem does not necessarily improve the total cost: better solutions to the static and deterministic subproblems do not translate into better solutions for the dynamic and stochastic underlying problem.

For the MSAC there are three parameters to consider: the number of scenarios, the number of sampled cargoes contained in each scenario, and the number of tabu search iterations used to solve each scenario. Figure 3 shows results where the number of scenarios is fixed to 15, and where the number of iterations is varied over the primary axis. The number of sampled cargoes in each scenario is measured relative to the number of known cargoes, so that less sampled cargoes are added when few cargoes are known. Results are reported where the number of scenarios is fixed to 15, and where the number of iterations is varied over the primary axis. The number of sampled cargoes in each scenario is measured relative to the number of known cargoes, so that less sampled cargoes are added when few cargoes are known. Results are reported where the number of scenarios is fixed to 30.

Figure 4 shows the effect of varying the number of scenarios used for each subproblem by the MSAC, using 100 % sampled cargoes. It appears that stable performance is reached once the number of scenarios reaches about 15. Also, if there is a potentially significant improvement to be had by increasing the number of iterations, this improvement is well hidden by the randomness of the solution instances.

Figures 5 and 6 illustrate the calibration of the BRH method. In Figure 5 the number of sampled cargoes is fixed to 50 %, and in Figure 6 the number of scenarios is fixed to 30. Again, it appears that a number of scenarios approaching 15 to 30 is sufficient to obtain good results. For the BRH the number of iterations take a different meaning, since one cannot tell in advance how many times each scenario is solved. However, using around 200 iterations per recalculation looks sufficient. Contrary to the MSAC, the BRH may benefit from a lower number of sampled cargoes per scenario. Although there
Fig. 1. Testing different time update mechanisms, with varying number of iterations per subproblem, for the MDH

Fig. 2. Testing the effect of varying the number of iterations per subproblem and the percentage of sampled cargoes per scenario in MSAC when using 15 scenarios per subproblem.

Fig. 3. Illustrating the effect of the number of scenarios in MSAC, using 100% sampled cargoes in each scenario.
is a trend that increasing the number of iterations per recalculation allows using a larger number of sampled cargoes, the best results are obtained using relatively few sampled cargoes.

Final settings used: For all methods we use event based time update. We simulate for one year, and to have a fair treatment of end-of-simulation effects, we solve the last part as a deterministic problem using 10,000 iterations of the tabu search. We use the solution from the previous subproblem as the initial solution for the new subproblem in all methods. For MDH we use 40,000 iterations of tabu search per subproblem. For BRH we use 30 scenarios with 25 % extra sampled cargoes and 200 iterations of tabu search to evaluate each decision. For MSAC we use 30 scenarios with 100 % extra sampled cargoes and 16,000 iterations per subproblem.

B. Main results

The main results are presented in Table I for instances with parcel load, and in Table II for full load instances. We first note the running times, which are low for the MDH and highest for the MSAC. Over all instances, the most computationally demanding instance requires slightly less than half an hour of CPU time per subproblem using the MSAC. This should be acceptable, given that planning is typically not performed more than once a day. We then observe that both MSAC and BRH are improving on MDH in terms of the total cost, by about 1–2% on the parcel load instances and only marginally on the full load instances. The cost reduction is broken down on different subsets of instances, and numbers in bold in the tables indicate that the cost saving is statistically significant on a 5 % level. So even if the improvement on full load instances is marginal, it is significant. Moreover, the cost saving is larger for instances with high demand, and for instances with fewer ports. It is interesting to notice that for MSAC the improvement is accompanied by a reduction of spot voyages, whereas for BRH it is accompanied by an increase of spot voyages. This indicates that the three methods produce solutions that are structurally different.

VI. CONCLUDING REMARKS

In this paper we have approached a dynamic and stochastic vehicle routing problem in industrial shipping. Three heuristics (MDH, BRH and MSA) based on different strategies are presented and described in detail. MDH is a deterministic algorithm based only on known information, and BRH and MSA are heuristics that take into account the stochastic information available concerning future cargo requests to design the current plan. In all three methods a tabu search procedure is used to solve the deterministic subproblems arising when replanning is performed.

To test the solution methods, a set of test instances is generated. Instances vary based on the fleet size (6, 12, or 24 vessels), the fleet composition (heterogeneous or homogeneous), the size of cargoes (full load or parcel load), the number of cargoes (high or low demand), and the number of ports (16 or 32). Each instance is simulated over a one year horizon. In the results we consider the total cost required to transport all cargoes, and also look at the number of spot voyages that had to be used. The three heuristics are calibrated on a small set of other instances, deciding the number of iterations required in the tabu search, the number of scenarios to be used, and the amount of sampled cargoes to be included in each scenario. Preliminary results show that the MDH is effective up to a certain point when increasing the number of iterations performed in the tabu search. However, when increasing the number of iterations further, the solutions of the deterministic replanning problems may improve without reducing the final costs observed by the simulation. Both the MSA and the BRH are able to make better use of increased computational effort, as increasing the number of scenarios and increasing the number of tabu search iterations used to solve the scenarios both improve the quality of the implemented solutions. Overall, the costs obtained using BRH and MSA are roughly between zero and five percent lower than when using MDH, but this varies according to the structure of the problem instances considered.

Although the cost savings obtained with the proposed methods are quite significative, there exist several lines of future research that could be interesting to explore. One is to consider more complex solution methods, introducing local search procedures to better exploit the stochastic information available based on the performance on all scenarios. Another relevant opportunity for further work is to extend this research to other modes of maritime transportation, as for instance tramp shipping. The development of specialized tools for several modes of maritime transportation problems that specifically take into account stochasticity would be very valuable.

ACKNOWLEDGEMENTS

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Fig. 4. Calibrating the number of scenarios per subproblem in BRH, using 50% sampled cargoes per scenario.

Fig. 5. Calibrating the number of tabu search iterations and the percentage of sampled cargoes in BRH, using 30 scenarios per subproblem.

TABLE I
SUMMARY OF RESULTS FOR 48 INSTANCES WITH PARCEL LOAD.

<table>
<thead>
<tr>
<th>Subset of instances</th>
<th>#I</th>
<th>Seconds per subproblem</th>
<th>Reduction in spot voyages</th>
<th>Cost saving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MDH</td>
<td>MSAC</td>
<td>BRH</td>
</tr>
<tr>
<td>Big fleet</td>
<td>16</td>
<td>26</td>
<td>1134</td>
<td>518</td>
</tr>
<tr>
<td>Medium fleet</td>
<td>16</td>
<td>6</td>
<td>241</td>
<td>45</td>
</tr>
<tr>
<td>Small fleet</td>
<td>16</td>
<td>1</td>
<td>35</td>
<td>3</td>
</tr>
<tr>
<td>High demand</td>
<td>24</td>
<td>17</td>
<td>780</td>
<td>340</td>
</tr>
<tr>
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<td>5</td>
<td>160</td>
<td>37</td>
</tr>
<tr>
<td>16 ports</td>
<td>24</td>
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<td>234</td>
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<tr>
<td>32 ports</td>
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<td>10</td>
<td>405</td>
<td>143</td>
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<tr>
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<td>470</td>
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TABLE II

Summary of results for 48 instances with full load.

<table>
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<th>Seconds per subproblem</th>
<th>Reduction in spot voyages</th>
<th>Cost saving</th>
</tr>
</thead>
<tbody>
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<td>MSAC</td>
<td>BRH</td>
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<td>13</td>
<td>1</td>
</tr>
<tr>
<td>Small fleet</td>
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<td>0</td>
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<td>23</td>
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</table>

TABLE III

Summary of results for 48 instances with parcel load and high spot voyage fees.

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<th>Subset of instances</th>
<th>#I</th>
<th>Seconds per subproblem</th>
<th>Reduction in spot voyages</th>
<th>Cost saving</th>
</tr>
</thead>
<tbody>
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<td>MDH</td>
<td>MSAC</td>
<td>BRH</td>
</tr>
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<td>Big fleet</td>
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<td>6</td>
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<td>166</td>
<td>39</td>
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<td>161</td>
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<td>12</td>
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<td>216</td>
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</table>

TABLE IV

Summary of results for 48 instances with parcel load and very high spot voyage fees.

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<th>#I</th>
<th>Seconds per subproblem</th>
<th>Reduction in spot voyages</th>
<th>Cost saving</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>MDH</td>
<td>MSAC</td>
<td>BRH</td>
</tr>
<tr>
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<tr>
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<td>882</td>
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<tr>
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<td>24</td>
<td>5</td>
<td>169</td>
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<td>48</td>
<td>12</td>
<td>526</td>
<td>222</td>
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and-price-and-cut method for ship scheduling with limited risk,” Transpor
tic for ship routing and scheduling,” Journal of the Operational
[14] H. Lo and M. McCord, “Adaptive ship routing through stochas-
tic ocean currents: General formulations and empirical results,” Transpor
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